**Unit 13 – Second Order Linear Differential Equations**

Notes: (1) It is possible to do this unit before systems (Unit 9), (2) This unit is more scaffolded than most other units.

Goals/Rationale

The intent of this unit is for students to develop the method of undetermined coefficients for second order differential equations with constant coefficients. After giving a few motivating examples for why second order DEs are interesting, students are given an opportunity to guess and test functions that are solutions. This is followed up by a series of problems that steps them through the conventional steps for the method of undetermined coefficients. The steps of the method are structured in a way that is intended to make sense for students, with less emphasis on reinventing the process. The unit is organized in two main parts. The first part develops the method to find the general solution to the homogeneous case and the second part develops the general solution to the standard collection of nonhomogeneous case.

In the homework problems students are introduced to the term characteristic equation and the method of variation of parameters. Problem 4 in the homework gives students a chance to reflect on and organize what one should guess for particular nonhomogeneous DEs. Also in the homework there are problems that relate to the motivating examples given at the very start of the unit.

**Pages 13.2-13.4 – Guess and test (leading to general solution for homogeneous case)**

Implementation Notes

*Problems 1-3* – Before starting the guess and test students should “read with meaning” four different fairly simple second order DE. Typical reading with meaning is something like, “x(t) is a function whose second derivative is equal to the negative of the function itself.” Reading with meaning will help them find two different solutions for each of the four DEs. Problem 2 continues the guess and test but with a slightly more complicated DE. Based on their guesses from problem 1 and the carefully chosen coefficients students are likely to correctly guess e^(-t). It can be useful, however, if students guess an incorrect function and relate back to reading with meaning. Students do not tend to think of multiplying their correctly guessed function by a constant and so problem 3 prompts them to do just this.

*Problem 4* – Now students have to try and guess a second solution function to the same DE from the previous problems. With some trial and error, they will eventually figure out that e^(-9t) works. Some may even anticipate or reinvent the approach of guessing e^kt, but we recommend NOT telling students about this approach since this is part of problem 6.

*Problem 5* – In this problem students get a chance to apply what they figured out about finding the general solution to systems of DEs to second order DEs. If students struggle with this you might ask them to think about how they formed the general solution for systems of DEs and use the same idea here. A good question to ask when they are finished is, “Why does it make sense that the sum of two solutions is again a solution?” A reasonable response is something like, “We know that if we plug in one of the solutions everything will add up to zero. Same with the other solution. So on the left hand side you get 0+0 , which still equals 0 on the right hand side.”

*Problems 6 & 7* – Problem 6 reviews and consolidates the steps taken in the previous problems. At the end of problem 6 students should have a systematic approach (that makes sense to them) for finding the general solution to the homogeneous equation. Problem 7 gives them a chance to practice the method on a different DE.

**Pages 13.5 – 13.10 – The nonhomogeneous case**

*Problems 8-16* – These problems lead students through the process of finding the general solution to standard nonhomogeneous cases. If pressed for time some of these can be assigned for homework or omitted if not necessary for your students.

* Problems 8-12 develop the approach for finding the general solution in when the right hand side of the nonhomogeneous equation is a constant or a sine or cosine.
* Problems 13-15 develop the approach for when the characteristic equation has complex roots
* Problem 16 takes up the case when the characteristic equation has a repeated root.

**Personal Reflections on Unit 13**